# Computer Vision II

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Machine Learning for Computer Vision
TU Dresden

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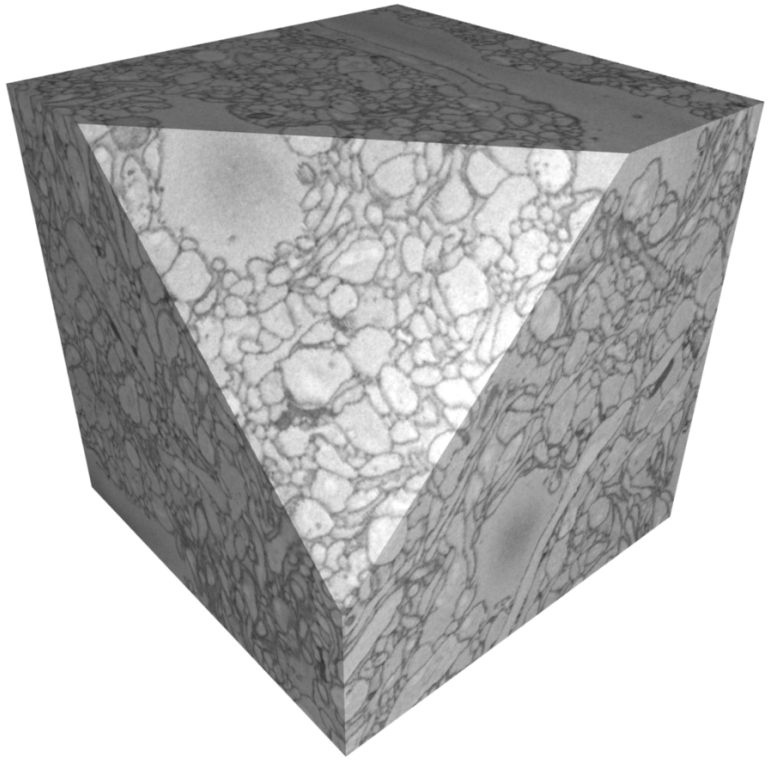
# Image decomposition

- So far, we have studied **pixel classification**, a problem whose feasible solutions define decisions at the pixels of an image
- Next, we will study **image decomposition**, a problem whose feasible solutions decide whether pairs of pixels are assigned to the same or distinct components of the image
- Image decomposition has applications where components of the image are indistinguishable by appearance (see next slide)



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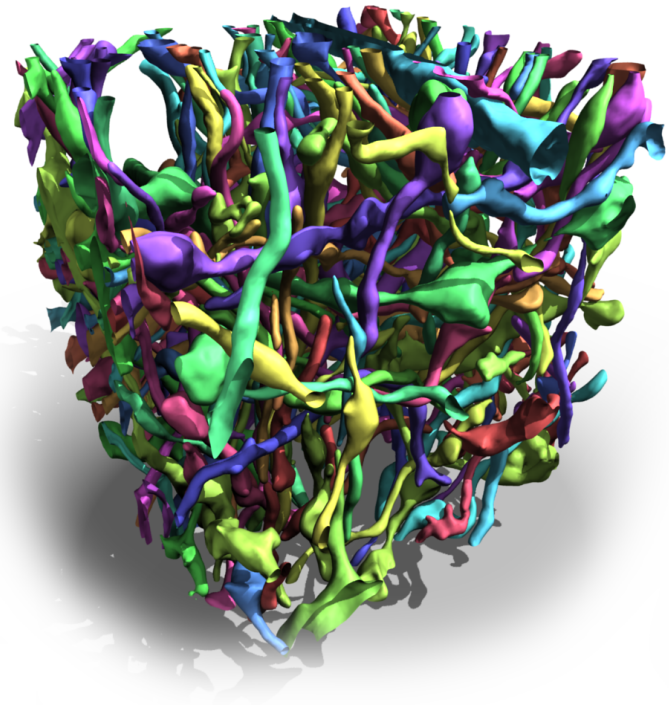
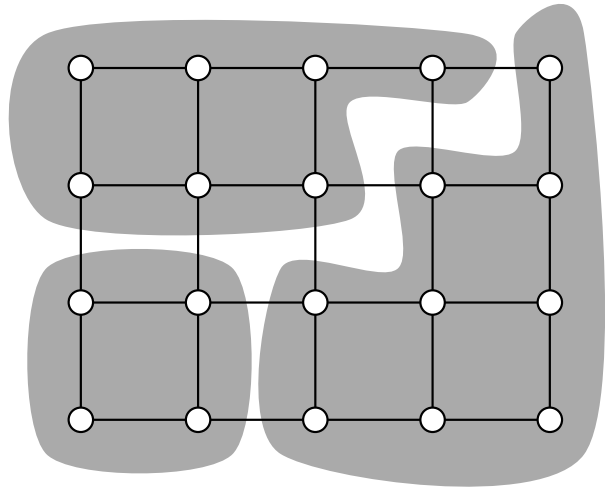


Volume Image (32 nm/voxel)
(Denk and Horstmann, 2004)

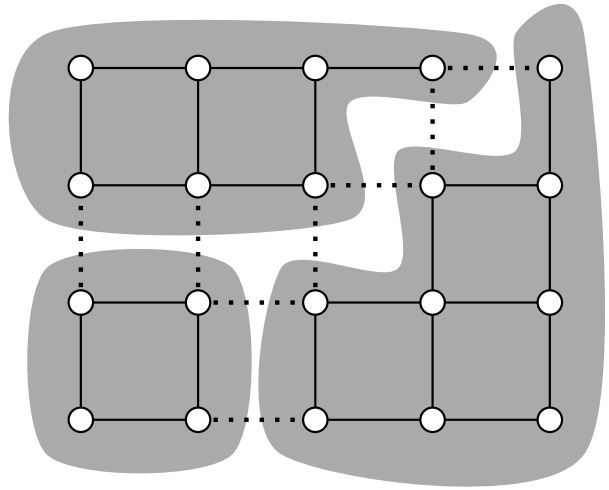
Decomposition
(Andres et al., 2012)

**Example:** The volume image on the top left taken by a Serial Block Face Scanning Electron Microscope shows cells that are indistinguishable by appearance. Decomposing such an image into individual cells is one challenge toward the ambitious goal of mapping the connectivity of nervous systems.



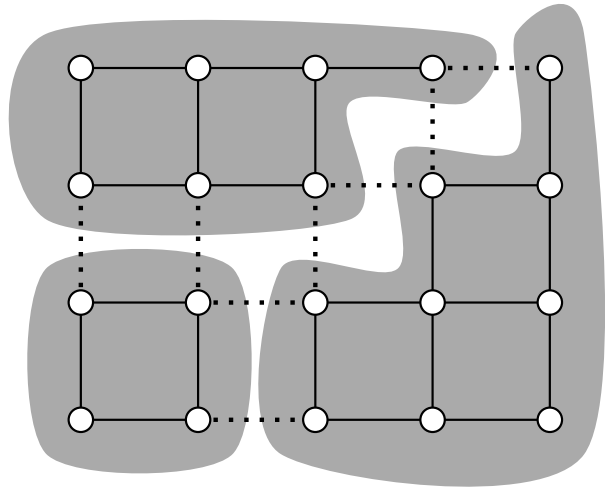
Decomposition of a graph $G = (V, E)$

- A mathematical abstraction of a decomposition of an image is a decomposition of the pixel grid graph.
- A decomposition of a graph is a partition of the node set into connected subsets (one example is depicted above in gray).



Decomposition of a graph $G = (V, E)$

- A decomposition of a graph is characterized by the set of edges that straddle distinct components (depicted above as dotted lines)
- Those subsets of edges are called **multicuts** of the graph



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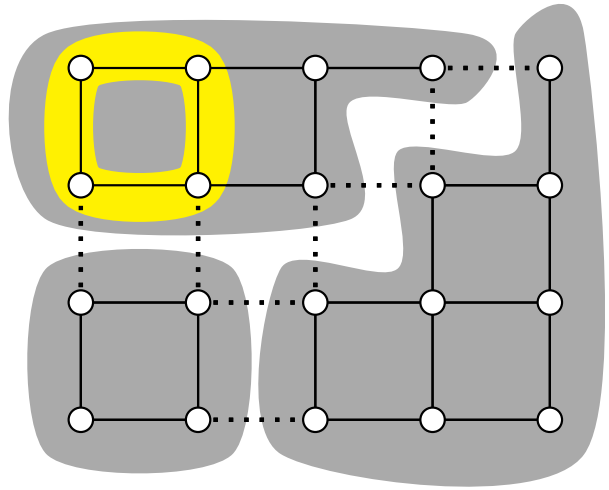
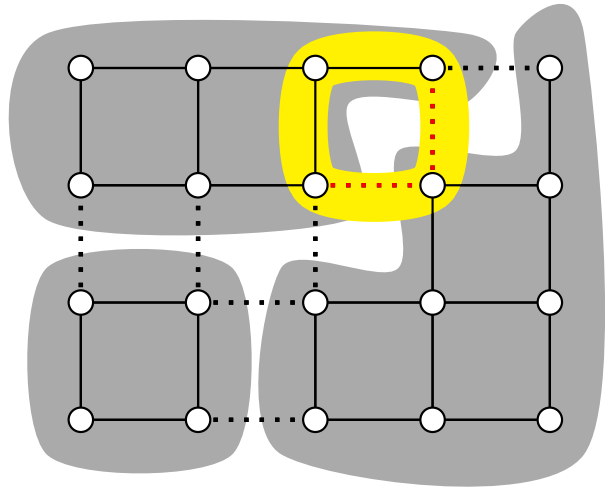


Multicut of a graph $G = (V, E)$

- The defining property of multicuts is that no cycle in the graph intersects with the multicut in precisely one edge



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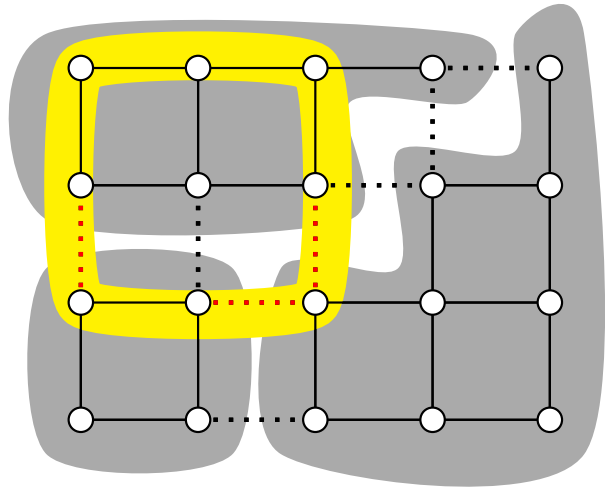

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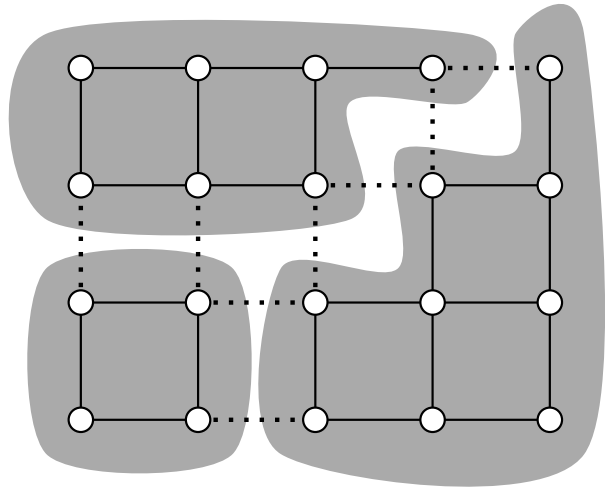
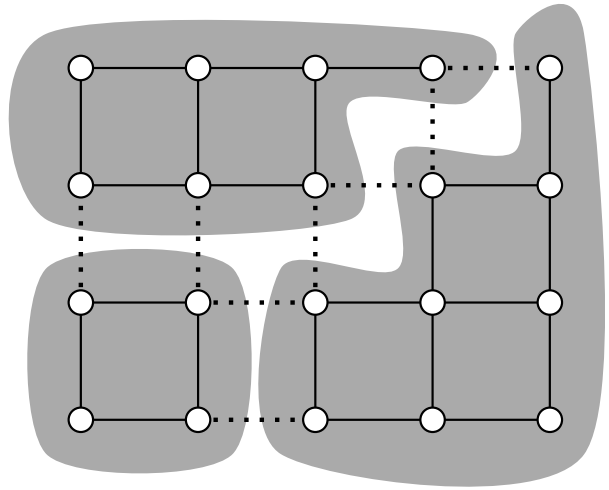


Multicut of a graph $G = (V, E)$

$$\text{multicuts}(G) := \{M \subseteq E \,|\, \forall C \in \text{cycles}(G) : \, |M \cap C| \neq 1\}$$



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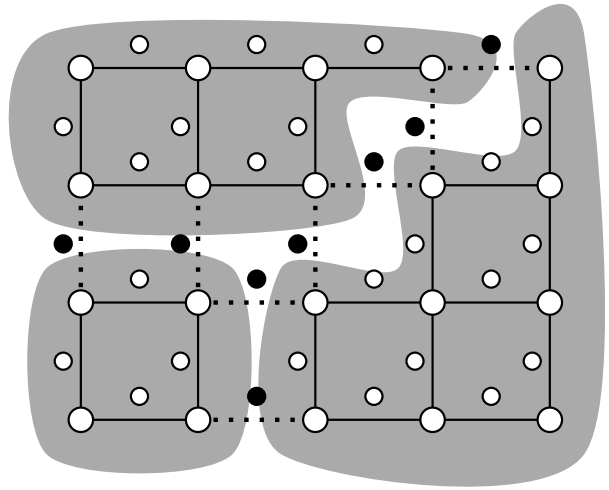
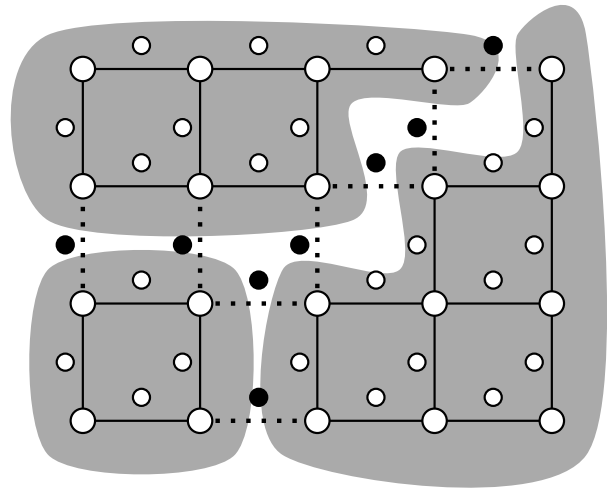


Multicut of a graph $G = (V, E)$

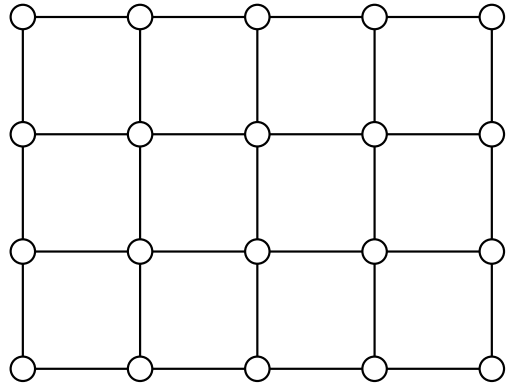
- The characteristic function $y\colon E \to \{0, 1\}$ of a multicut $y^{-1}(1)$ can be used to encode the decomposition induced by the multicut in an $|E|$-dimensional 01-vector
- For any $e \in E$, $y_e = 1$ indicates that an edge is cut, straddling distinct components



Multicut of a graph $G = (V, E)$

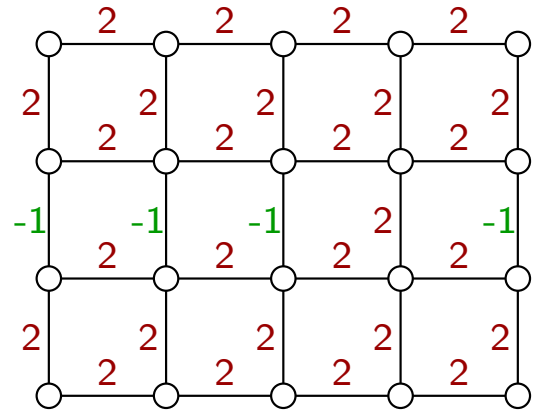
- The set of the characteristic functions of all multicuts of $G$:

$$Y_G := \left\{ y : E \to \{0,1\} \;\middle|\; \forall C \in \mathsf{cycles}(G)\, \forall e \in C :\; y_e \leq \sum_{f \in C \setminus \{e\}} y_f \right\}$$



Graph $G = (V, E)$

- An instance of the image decomposition problem is given by a graph $G = (V, E)$ and, for every edge $e = \{v, w\} \in E$, a (positive or negative) cost $c_e \in \mathbb{R}$ that is payed iff the incident pixels $v$ and $w$ are put in distinct components
- Such costs are often estimated from examples using machine learning techniques




Graph $G = (V, E)$. Edge costs $c : E \to \mathbb{R}$

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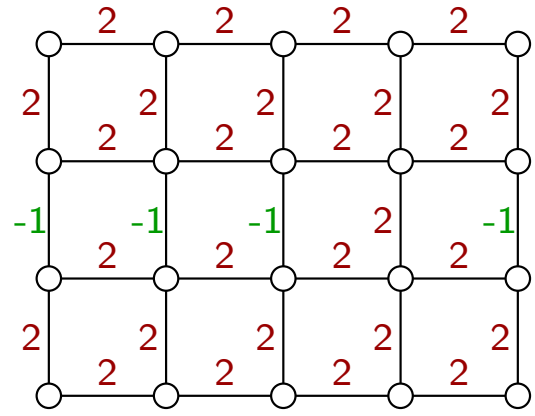
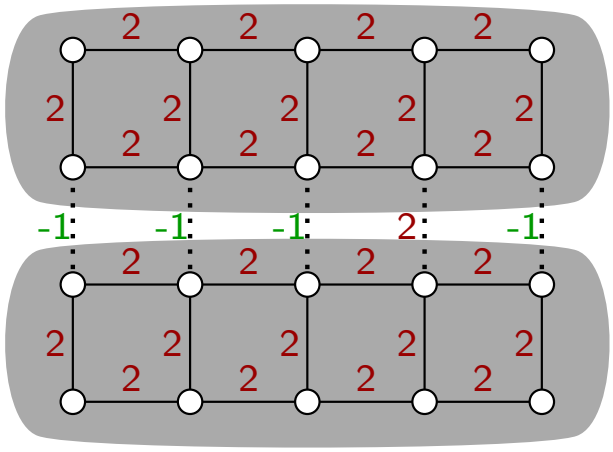



Graph $G = (V, E)$. Edge costs $c : E \to \mathbb{R}$

- Image decomposition problem:

$$\min_{y \in Y_G} \sum_{e \in E} c_e \, y_e$$

- The optimal solution is shown in the next slide

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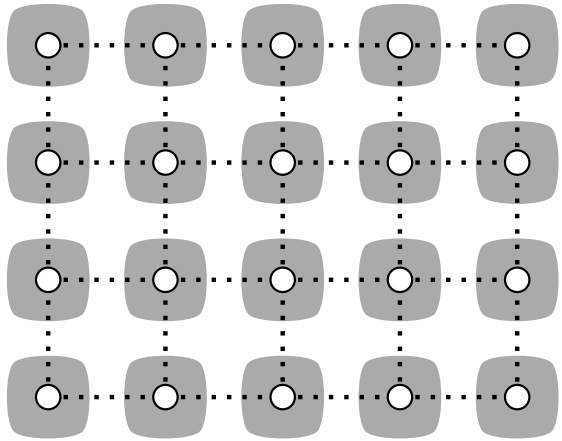
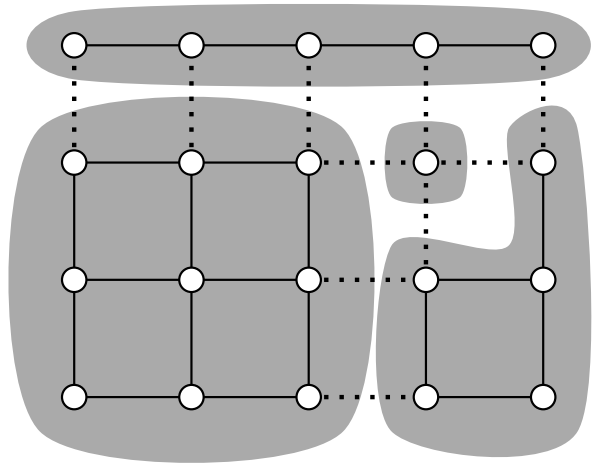
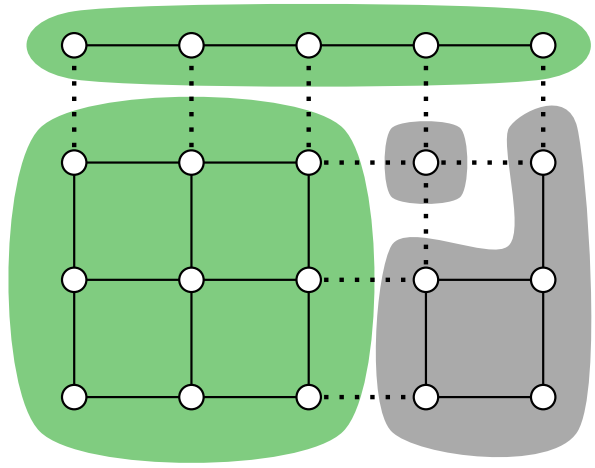


- One technique for finding feasible solutions to an image decomposition problem is **local search**.
- Starting from the finest decomposition into singleton components (depicted above), we greedily join neighboring components as long as this improves the cost (see next slide).



- Once no joining of neighboring components further reduces the cost, we consider all pairs of neighboring components (depicted in green) and all nodes at the shared boundary (depicted in black) and all possibilities of moving nodes from one component to the other.
- The procedure is iterated until no such transformation further reduces the cost



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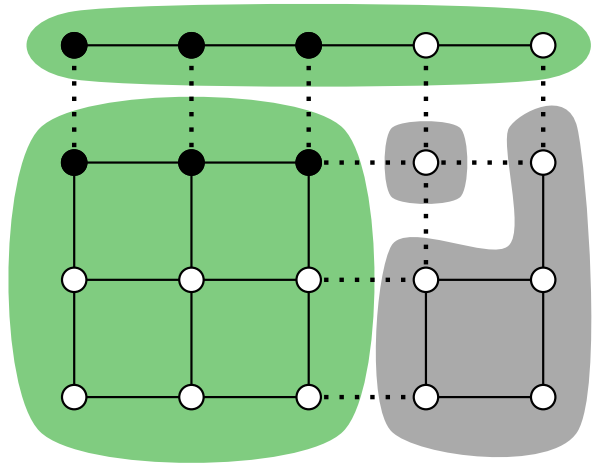

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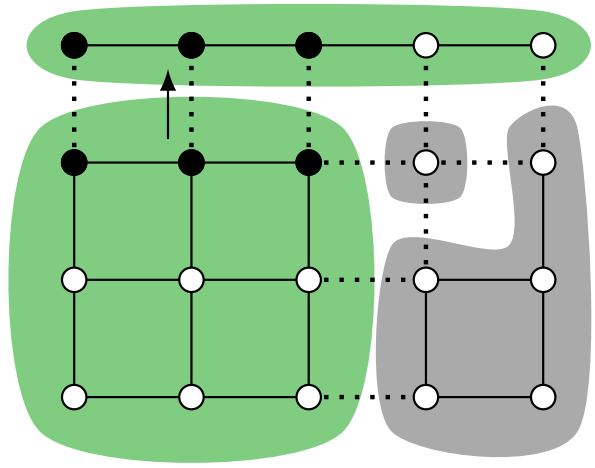

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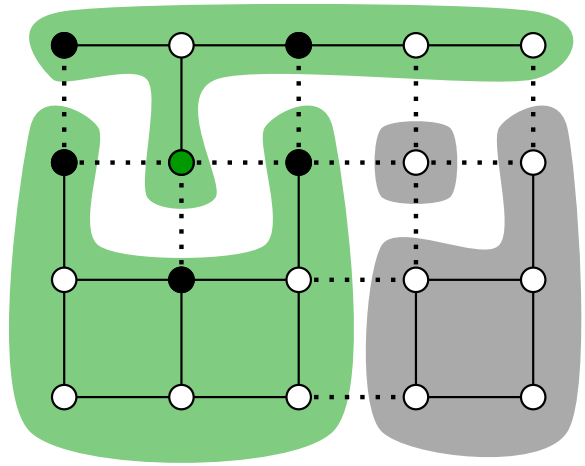

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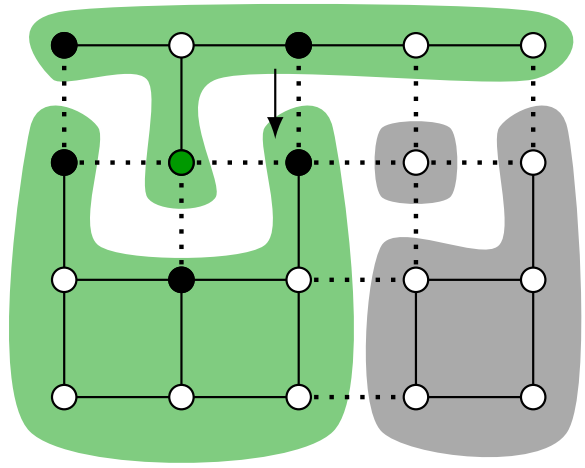

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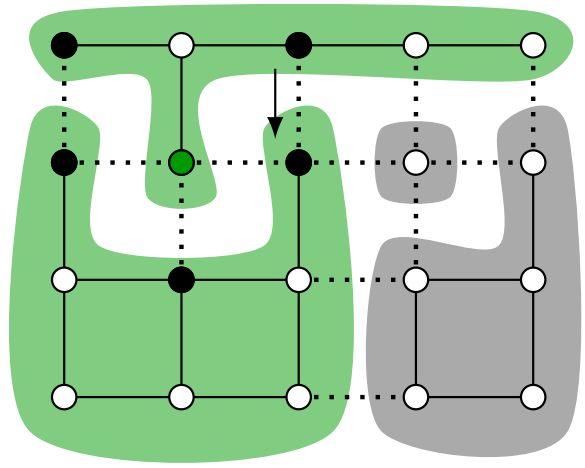

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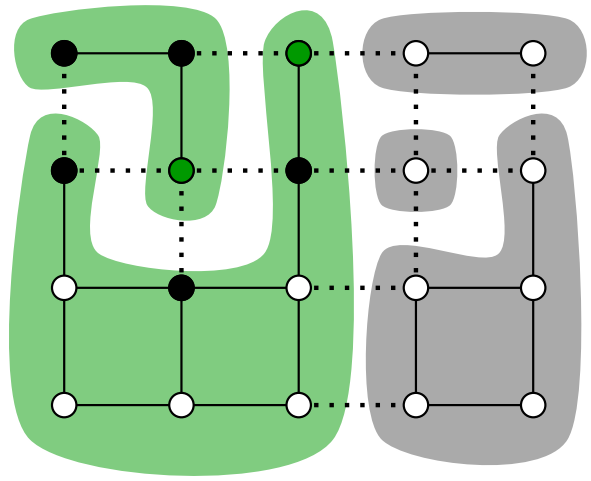

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**Self-study:**

- Implement a local search algorithm for the image decomposition problem
- Define costs for the colors of two pixels that are "large and positive if the colors are similar, and large and negative if the colors are dissimilar"
- Apply your implementation of local search to the image from the previous lectures and your cost function
- Discuss your findings