Computer Vision I

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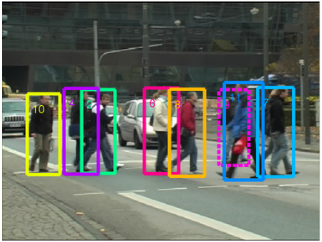
Machine Learning for Computer Vision
TU Dresden

https://mlcv.cs.tu-dresden.de/courses/25-winter/cv1/

Winter Term 2025/2026

Multiple Object Tracking

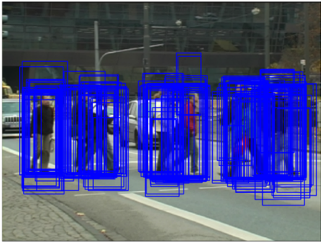
$\mapsto$



Video

Tracks

Multiple Object Tracking

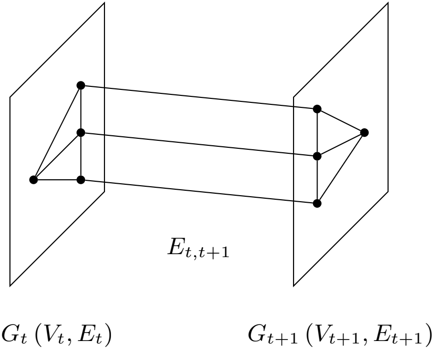


Video

$\mapsto$

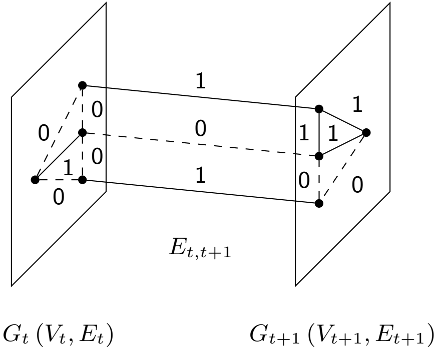
Tracks

Tang et al. 2016, 2017

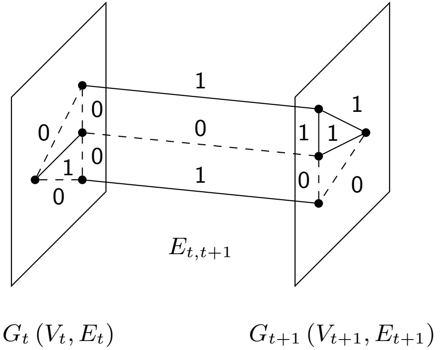


- Let $t \in \mathbb{N}$ and let $G\left(V, E\right)$ be a graph with

$$V = \bigcup_{t=0}^{n-1} V_t \quad \text{and} \quad E = \bigcup_{t=0}^{n-1} E_t \cup \bigcup_{t=0}^{n-2} E_{t,t+1}$$

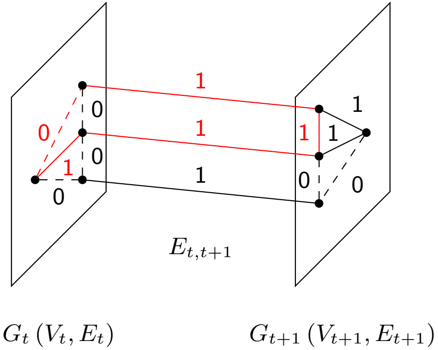


- Let $y \in \{0,1\}^E$.
- For any $t \in n$ and any $\{v,w\} = e \in E_t$, let $y_e = 1$ indicate that v and w belong to the same object.
- For any $t \in n-1$ and any $\{v,w\} = e \in E_{t,t+1}$, let $y_e = 1$ indicate that v and w belong to the same (object-)track.



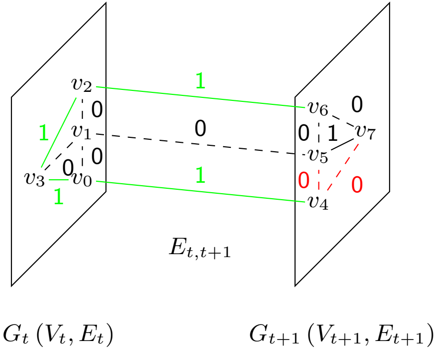
A feasible solution to the multiple object tracking problem is a $y \in \{0,1\}^E$ that satisfies three classes of constraints (discussed below):

1. Clustering constraints
2. No-split constraints
3. No-join constraints



Clustering constraints:

$$\forall C \in \mathsf{cycles}\,(G)\,\forall e \in C\colon \quad 1 - y_e \leq \sum_{e' \in C \setminus \{e\}} (1 - y_{e'}) \tag{1}$$

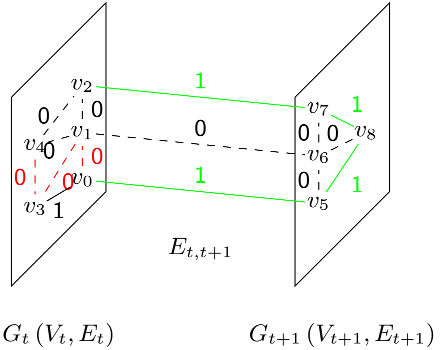


No-split constraints:

$$\forall t \in n-1\ \forall \{u,v\} \in \binom{V_{t+1}}{2}$$
$$\forall C \in \text{uv-cuts}\,(G_{t+1})\ \ \forall P \in \text{uv-paths}\,((V_t \cup V_{t+1}, E_t \cup E_{t,t+1})):$$

$$1 - \sum_{e \in P} (1 - y_e) \leq \sum_{e \in C} y_e \tag{2}$$

Multiple Object Tracking



No-join constraints:

$$\forall t \in n-1 \ \forall \{u, v\} \in \binom{V_t}{2}$$
$$\forall C \in \text{uv-cuts}(G_t) \ \forall P \in \text{uv-paths}((V_t \cup V_{t+1}, E_{t+1} \cup E_{t,t+1})):$$
$$1 - \sum_{e \in P} (1 - y_e) \leq \sum_{e \in C} y_e \tag{3}$$

Definition. For any t and G as defined above, and for any $c \in \mathbb{R}^E$, the instance of the **naïve multiple object tracking problem** wrt. G, t and c is

$$\begin{aligned} \max_{y \in \{0,1\}^E} \quad & \sum_{e \in E} c_e y_e \\ \text{subject to} \quad & (1)\,,(2)\,,(3) \end{aligned}$$

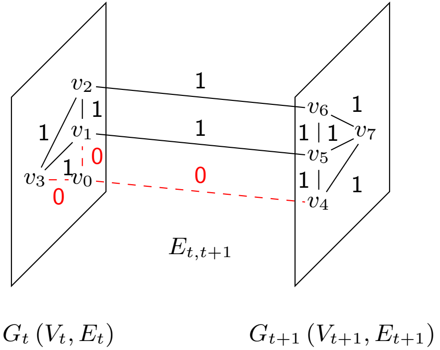
Remark: Any y defined by a set of node-disjoint paths satisfies (1), (2) and (3). Such a y can be used as an initial solution for a local search algortihm.

Next, we will discuss generalizations of the naïve multiple object tracking problem:

- Node variables and constraints
- Track termination variables and constraints
- Track creation variables and constraints

Node variables and constraints: Let $z \in \{0,1\}^V$ and $c' \in \mathbb{R}^V$.

$$\forall e \in E \ \forall v \in e\colon \quad y_e \leq z_v \tag{4}$$



Track termination variables and constraints: Let $x \in \{0,1\}^V$ such that

$$\forall t \in n-1 \;\forall u \in V_t \;\forall C \in \mathsf{u}V_{t+1}\text{-}\mathsf{cuts}((V_t \cup V_{t+1}, E_t \cup E_{t,t+1})):$$

$$z_u - \sum_{e \in C} y_e \leq x_u \tag{5}$$

Track creation variables and constraints: Let $x' \in \{0,1\}^V$ such that

$$\forall t \in n-1 \;\forall u \in V_{t+1} \;\forall C \in V_t\text{u-cuts}((V_t \cup V_{t+1}, E_{t,t+1} \cup E_{t+1}))$$

$$z_u - \sum_{e \in C} y_e \leq x'_u \tag{6}$$

Definition. For t and G as defined above, for any $c \in \mathbb{R}^E$, any $c' \in \mathbb{R}^V$ and any $c'', c''' \in (\mathbb{R}_0^-)^V$, the instance of the **multiple object tracking problem** wrt. t, G, c, c', c'', c''' is:

$$\begin{aligned} \max_{y \in \{0,1\}^E, x, x', z \in \{0,1\}^V} \quad & \sum_{e \in E} c_e y_e + \sum_{v \in V} c'_v z_v + \sum_{v \in V} c''_v x_v + \sum_{v \in V} c'''_v x'_v \\ \text{subject to} \quad & (1)\,, (2)\,, (3)\,, (4)\,, (5)\,, (6) \end{aligned}$$

One may further generalize the multiple object tracking problem so as to capture long-range connectedness:

For any $F \subseteq \binom{V}{2} \setminus E$:

$$\forall \{u,v\} \in F \ \forall P \in vw\text{-}\mathsf{paths}\,(G):$$

$$1 - y_{\{u,v\}} \leq \sum_{e \in P} (1 - y_e)$$

$$\forall \{u,v\} \in F \ \forall C \in vw\text{-}\mathsf{cuts}\,(G):$$

$$y_{\{u,v\}} \leq \sum_{e \in C} y_e$$