

Machine Learning 1 – Exercise 2

Machine Learning for Computer Vision
TU Dresden

Deciding with binary decision trees (BDTs)

- a) Construct two distinct BDTs such that the function defined by these BDTs equals the function g defined in Tab. 1.
- b) Let V be a finite, non-empty set. Define an algorithm that takes any disjoint sets $A, B \subseteq \{0, 1\}^V$ as the input and outputs a V -variate, $\{0, 1\}$ -valued BDT θ such that the function f_θ defined by this BDT has the properties $f_\theta(A) = 0$ and $f_\theta(B) = 1$.
- c) Prove correctness of your algorithm.
- d) Prove the following universality lemma: For any finite, non-empty set V and any $f: \{0, 1\}^V \rightarrow \{0, 1\}$, there exists a V -variate, $\{0, 1\}$ -valued BDT defining f .

Table 1: Defined by the value table below is a Boolean function $g: \{0, 1\}^V \rightarrow \{0, 1\}$ with $V = \{0, 1, 2\}$.

x_0	x_1	x_2	$g(x)$
0	0	0	0
1	0	0	1
0	1	0	0
1	1	0	1
0	0	1	1
1	0	1	0
0	1	1	1
1	1	1	0