

Machine Learning 1 – Exercise 1

Machine Learning for Computer Vision
TU Dresden

Deciding with disjunctive normal forms (DNFs)

- Let $V = \{0, 1, 2, 3\}$. State the V -variate DNF defined by $\theta = \{(\emptyset, \{0\}), (\{0\}, \{3\}), (\{0, 3\}, \{1, 2\})\}$, its length and its depth.
- State two distinct DNFs such that the function defined by these DNFs equals the function g defined in Tab. 1 below.
- How many distinct DNFs in $n = |V|$ variables exist?
- Prove the following universality property: For any finite, non-empty set V and any $f: \{0, 1\}^V \rightarrow \{0, 1\}$, there exists a V -variate DNF defining f .

Table 1: Defined by the value table below is a Boolean function $g: \{0, 1\}^V \rightarrow \{0, 1\}$ with $V = \{0, 1, 2\}$.

| x_0 | x_1 | x_2 | $g(x)$ |
|-------|-------|-------|--------|
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 0 |